Lecturered By Dr. H.K. Yadav:
Deptt. of Mathematics Marwari College, DBU
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Real Analysis (Comparision Test)

Q. State and prove comparision Test.

Soln: Statement: → If Σu_n and Σv_n be two positive term series and after some particular term $\frac{u_n}{u_{n-1}} < \frac{v_n}{v_{n-1}}$, then Σu_n converges provided that Σv_n converges.

proof: Let the series be

$$u_1 + u_2 + u_3 + \cdots + u_n + \cdots \text{ to } \infty,$$

and $$v_1 + v_2 + v_3 + \cdots + v_n + \cdots \text{ to } \infty.$$

It is given that

$$\frac{u_2}{u_1} < \frac{v_2}{v_1}$$

$$\frac{u_3}{u_2} < \frac{v_3}{v_2}$$

$$\cdots\cdots$$

$$\frac{u_n}{u_{n-1}} < \frac{v_n}{v_{n-1}}$$

Multiplying all these inequalities, we get

$$\frac{u_2}{u_1} \cdot \frac{u_3}{u_2} \cdots \frac{u_n}{u_{n-1}} < \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} \cdots \frac{v_n}{v_{n-1}}$$

$$\Rightarrow \frac{u_n}{u_1} < \frac{v_n}{v_1} \text{ or } u_n < \left(\frac{u_1}{v_1}\right) v_n$$

Let $\frac{u_1}{v_1} = K$. Then $u_n < K v_n$ or $\Sigma u_n < K \Sigma v_n$

If Σv_n converges, then Σu_n also converges.

Q. 2. If $\sum u_n$ and $\sum v_n$ be two positive term series and after some particular term $\frac{u_n}{u_{n-1}} > \frac{v_n}{v_{n-1}}$, then $\sum u_n$ diverges provided that $\sum v_n$ diverges.

Proof: Let the series be

$$u_1 + u_2 + u_3 + \ldots + u_n + \ldots \text{ to } \infty$$

and $$v_1 + v_2 + v_3 + \ldots + v_n + \ldots \text{ to } \infty$$

It is given that $\frac{u_2}{u_1} > \frac{v_2}{v_1}$

$$\frac{u_3}{u_2} > \frac{v_3}{v_2}$$

$$\cdots \quad \cdots$$

$$\frac{u_n}{u_{n-1}} > \frac{v_n}{v_{n-1}}$$

Multiplying all these inequalities, we get

$$\frac{u_2}{u_1} \cdot \frac{u_3}{u_2} \cdots \frac{u_n}{u_{n-1}} > \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} \cdots \frac{v_n}{v_{n-1}}$$

$$\Rightarrow \frac{u_n}{u_1} > \frac{v_n}{v_1} \quad \text{or,} \quad u_n > \left(\frac{u_1}{v_1}\right) v_n$$

Let $\frac{u_1}{v_1} = K$

Then, $u_n > K v_n$ or, $\sum u_n > K \sum v_n$

If $\sum v_n$ is divergent, then $\sum u_n$ is also divergent.